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LETTER TO THE EDITOR

The paper by George A. Keramidas: A generalized variational formulation for convective heat transfer, *Int. j. numer. methods fluids*, **1**, 305–322 (1981), deserves comment. The paper first presents a method due to Biot[1–5] without making reference to any of Biot's contributions. Biot introduces a heat flow vector and solves for it instead of temperature when solving the energy equation. This approach has been shown[6–8] to be equivalent to a Galerkin method and has no advantages. To quote from Reference 8,

> This quasi-variational principle does reproduce the original equations, but it is unduly complicated. Note that certain equations must be solved exactly. Since it is usually easy to make good guesses for trial functions for the temperature, we must then solve a partial differential equation exactly before proceeding to apply the quasi-variational principle. For two- or three-dimensional problems this is a serious disadvantage. Since no variational integrals are made stationary, that common advantage of variational principles is lost. The Galerkin method has been shown to give the same approximate solution, if formulated in terms of the same variables, H and h, so that the quasi-variational principle gains no advantage in application.[6,7] The Galerkin method applied in terms of temperature alone, without the introduction of "total heat" and "heat flow vectors," is much simpler and certainly would be preferred in computations. The examples solved by Biot do provide an illustration of approximate solutions useful in a variety of one-dimensional problems.[5]
>
> Biot attempts to interpret his quasi-variational principle in terms of minimum rate of entropy production. As shown elsewhere[7] the interpretation applies only to the approximate solution, not necessarily to the exact solution so that the physical meaning of the principle is somewhat vacuous.

Unfortunately Keramidas ignored this literature as well.

The new results in this paper are just an illustration of Biot's method applied to the one-dimensional convective diffusion equation. What the author could have done, but didn't, was compare solutions obtained by two methods: (1) expanding temperature in a finite element basis and solving the convective–diffusion equation and (2) using Biot's idea to solve for the heat flow vector. Based on the results in the paper Biot's method appears to have nothing to recommend it; it certainly does not 'suppress the wiggles'.

Finally, note that one can expand both the temperature, θ, and the heat flux, $\mathbf{q}$, in a finite element basis, and apply the Galerkin method to the energy equation, $\nabla \cdot \mathbf{q} = —$, coupled with the constitutive equation $\mathbf{q} = -k\nabla\theta$. Results on use of $(\theta, \mathbf{q})$ versus (θ alone) would be useful information for the Galerkin method as well as Biot's method.

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